Midterm Exam II: MAT 310

Instructions: Complete all problems below. You may not use calculators or other electronic devices, including cell phones. Show all of your work. **Be sure to write your name and student ID on each page that you hand in.**

1.(14pts) Determine the values of α for which the following system of linear equations is consistent, and write down the general solution.

$$3x_1 + 2x_2 + 3x_3 - 2x_4 = 1$$
$$x_1 + x_2 + x_3 = \alpha$$
$$x_1 + 2x_2 + x_3 - x_4 = 2$$

2.(15pts) Let
$$A = \begin{pmatrix} 4 & 3 & 1 & 2 \\ 1 & 9 & 0 & 2 \\ 8 & 3 & 2 & -2 \\ 4 & 3 & 1 & 1 \end{pmatrix}.$$

a) Calculate the determinant of A using any method that you know.
b) What is the determinant of $-2A$?

3.(13pts) Let A and B be $n \times n$ matrices such that $AB = -BA$. Prove that if n is odd then A or B is not invertible.

4.(13pts) Determine if the following matrix is diagonalizable and justify your answer. If so, find an invertible matrix Q and a diagonal matrix D such that $A = QDQ^{-1}$.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5.(15pts) Solve the following system of differential equations with initial condition:

$$\begin{aligned} x_1' &= 3x_1 - 2x_2, & x_1(0) &= 1, \\ x_2' &= 2x_1 - 2x_2, & x_2(0) &= -1. \end{aligned}$$

6.(15pts) Let A be an $n \times n$ matrix. Prove that the set $\{I, A, \ldots, A^{n-1}, A^k\}$ is linearly dependent for every integer $k \geq n$.

7.(15pts) Compute the Jordan canonical form J of the matrix A in problem 4 above. In addition, find an invertible matrix Q such that $A = QJQ^{-1}$.